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A Multiscale Approach to the Design and Analysis of WEAV3D Lattice Structures

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WEAV3D®

ALTAIR

Introduction to WEAV3D Lattice Structures

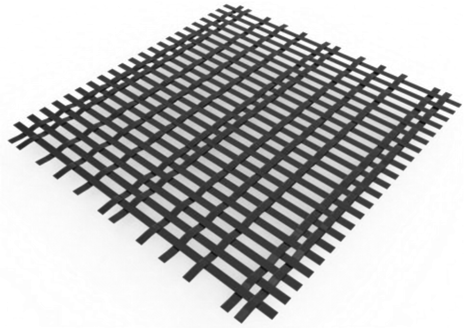
BE BOUNDLESS

WEAV3D®

WEAV3D Lattice Structures

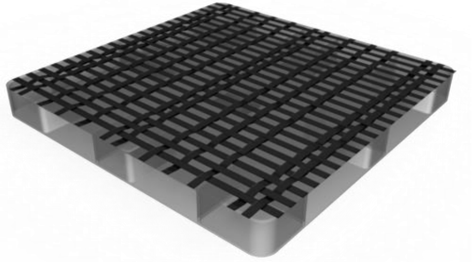
Thermoplastic Prepreg Tape Woven into Lattice Structures Embedded in Thermoplastic Structural Components to Provide Cost/Weight/Strength Efficient Reinforcement

Lattice Structure
(Thermoplastic Prepreg Tape)



Lattice Structure Embedded in a Precast Polymer Concrete Trench

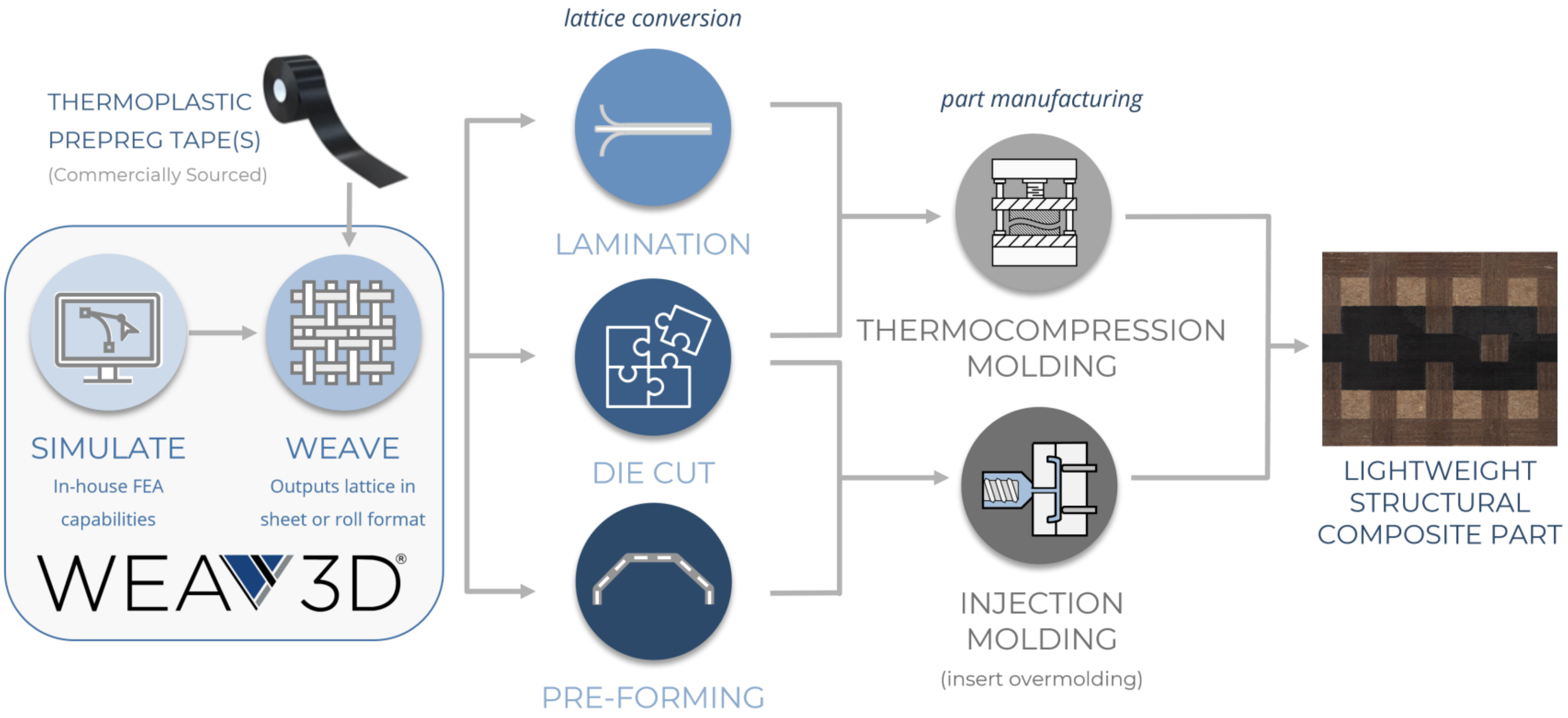
Lattice Structure Embedded in a Thermoplastic Pallet



Lattice Structure Embedded in a Nature Fiber Nonwoven Mat

WEAV3D

Rebar for Plastics ®



WEAV3D®

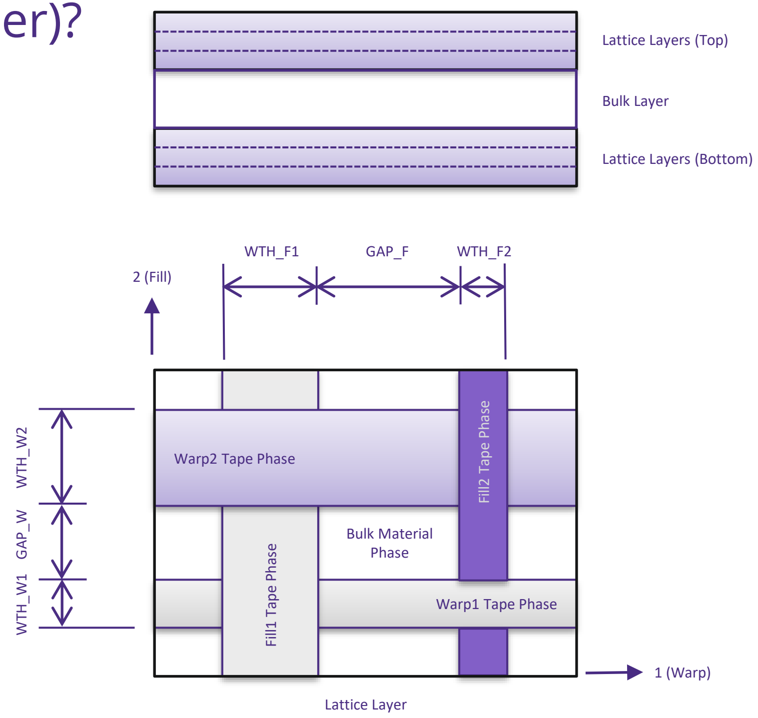
Design and Analysis Challenges

How do you design and analyze something like this?
(Example: Automotive Beltline Stiffener)

WEAV3D

Design and Analysis Challenges

- What Warp Tape Material (Carbon, Glass, other)?
- What Fill Tape Material (Carbon, Glass, other)?
- What Tape Width (Warp & Fill)?
- What Tape Gap (Warp & Fill)?
- How Many Lattice Layers (Top & Bottom)
- What Bulk Material?



Even if we know these things, how to analyze structures made from these lattice structures?

Design and Analysis Challenges

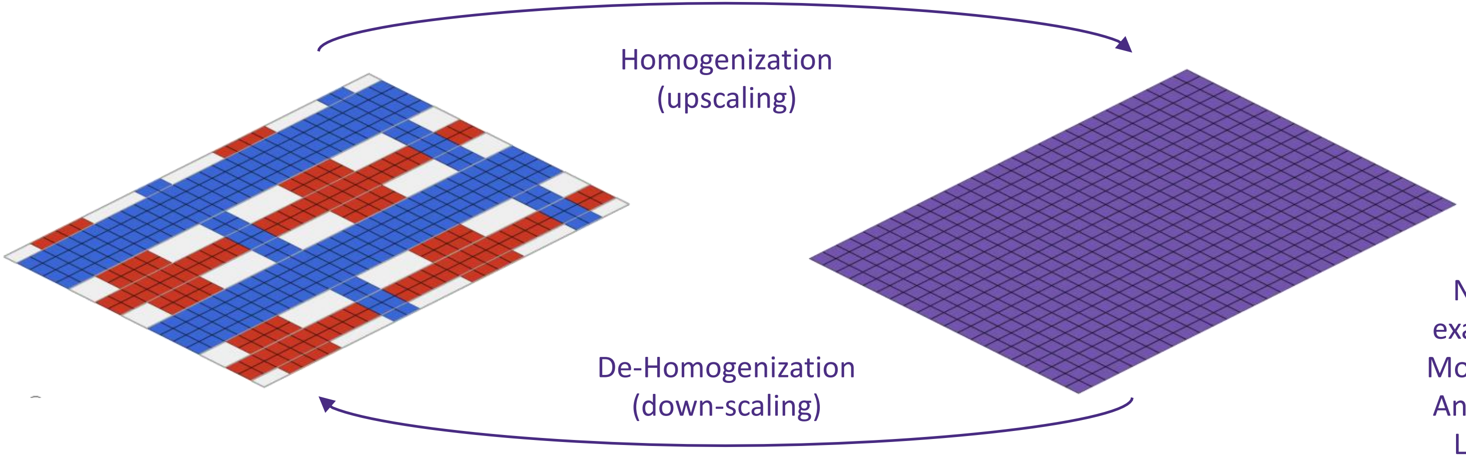
Discrete Modeling

VS

-Explicitly model features
-typically most Accurate Results
-can be Difficult to Model
-can be Computationally Expensive

Multiscale Modeling

-Implicitly model features
-can achieve Accurate Results
-significantly Easier Modeling
-can be Computationally Efficient



Note: A great early example of Multiscale Modeling is Composite Analysis with Classical Lamination Theory

Introduction to a Proposed Multiscale Solution

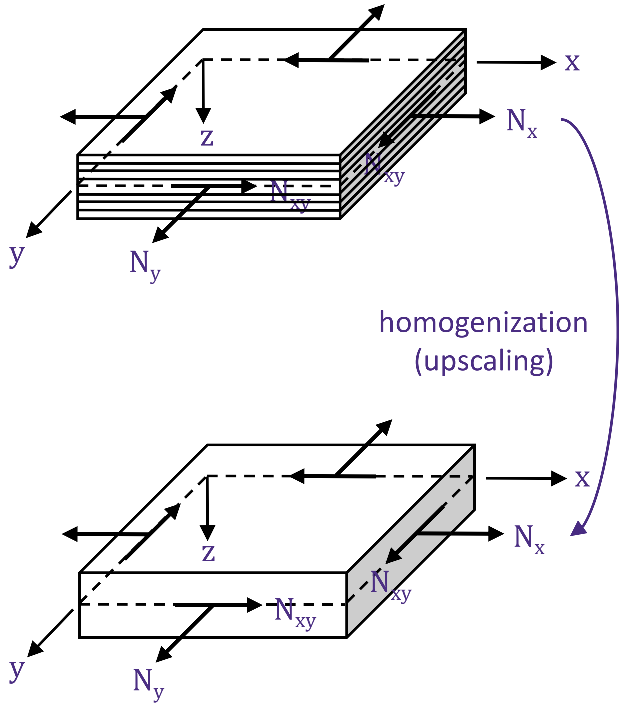
Proposed Multiscale Solution

“All truths are easy to understand once they are discovered; the point is to discover them.” Galileo Galilei

We are simply connecting already discovered puzzle pieces to assemble a new puzzle picture

Puzzle Piece #1
Classical Lamination Theory

One of the earlier forms of Multiscale Analysis…



starting from the strain of the k^{th} ply

$$(\varepsilon)_k = (\varepsilon^o) + z_k(\kappa)$$

$$(\sigma)_k = [\bar{Q}]_k\,((\varepsilon)_k - (\alpha)_k \Delta T)$$

$$(N) = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} (\sigma)_k\, dz$$

$$(N) = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} [\bar{Q}]_k\,((\varepsilon^o) + z_k(\kappa) - (\alpha)_k \Delta T)\, dz$$

$$(N) = [A](\varepsilon^o) + [B]\,(\kappa) - (N_t)$$

the homogenized stiffnesses of a laminated plate are;

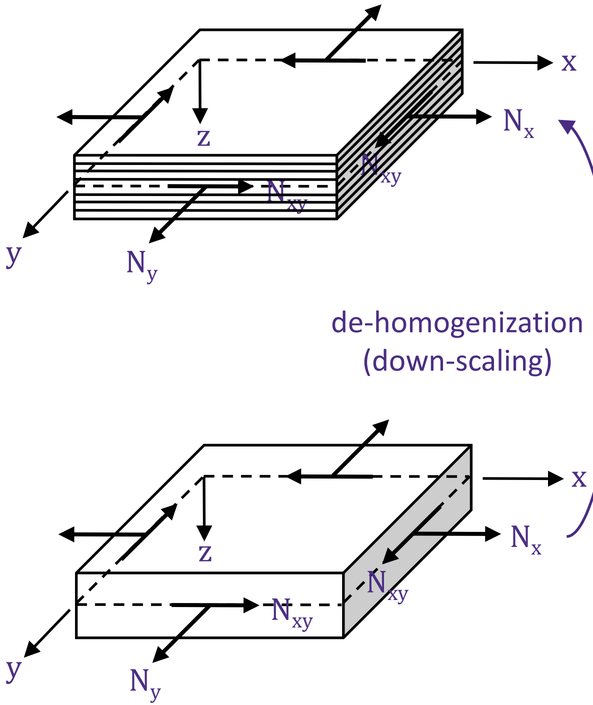
$$[A] = \sum_{k=1}^{n} [\bar{Q}]_k\,(z_k - z_{k-1})$$

$$[B] = \frac{1}{2}\sum_{k=1}^{n} [\bar{Q}]_k\,(z_k^2 - z_{k-1}^2)$$

$$(N_t) = \Delta T \sum_{k=1}^{n} [\bar{Q}]_k\,(\alpha)_k\,(z_k - z_{k-1})$$

Puzzle Piece #1
Classical Lamination Theory

One of the earlier forms of Multiscale Analysis…



de-homogenization
(down-scaling)

the stresses of the k^{th} ply are;

$$(\sigma)_k = [\bar{Q}]_k \left((\varepsilon)_k - (\alpha)_k \Delta T\right)$$

$$(\varepsilon)_k = (\varepsilon^o) + z_k(\kappa)$$

$$\begin{pmatrix}\varepsilon^o \\ \kappa\end{pmatrix} = \begin{bmatrix} a & b \\ b & d \end{bmatrix} \left(\begin{pmatrix} N \\ M \end{pmatrix} + \begin{pmatrix} N_t \\ M_t \end{pmatrix} \right)$$

$$\begin{pmatrix} N \\ M \end{pmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{pmatrix}\varepsilon^o \\ \kappa\end{pmatrix} - \begin{pmatrix} N_t \\ M_t \end{pmatrix}$$

starting from the solution to the homogenized laminated plate equations

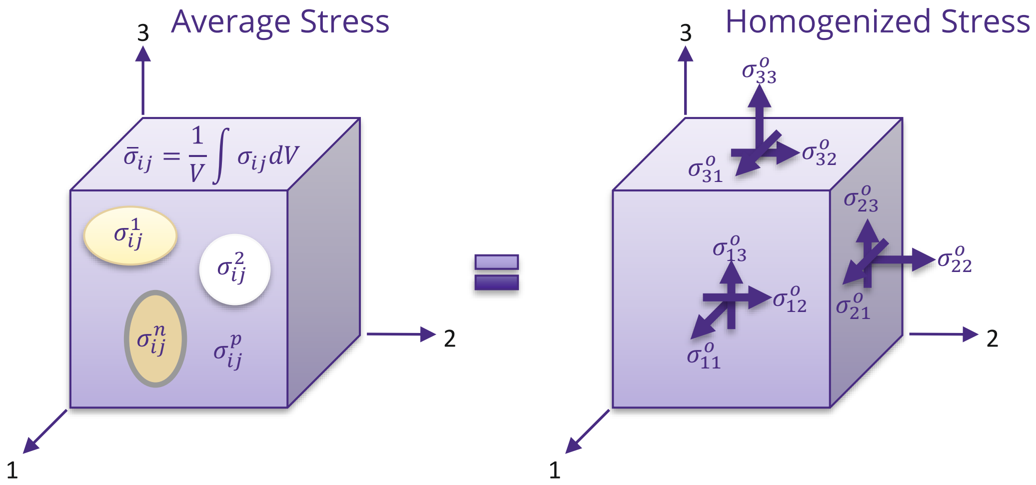
Puzzle Piece #2
Phase Average Theory

Hill, R., “The Mathematical Theory of Plasticity”, Oxford University Press, 1950

Hashin, Z., “Theory of Fiber Reinforced Materials”, NASA CR-1974, March 1972

The average stress ($\bar{\sigma}$) of all phases (p) is the homogenized stress (σ^o)

Fundamental result for multiscale applications



$$\bar{\sigma}_{ij} = \frac{1}{V}\int \sigma_{ij}dV = \sum_{p=1}^{n} \sigma^p_{ij} V^p = \sigma^o_{ij}$$

$$\bar{\varepsilon}_{ij} = \frac{1}{V}\int \varepsilon_{ij}dV = \sum_{p=1}^{n} \varepsilon^p_{ij} V^p = \varepsilon^o_{ij}$$

where

$$V^p = \frac{Volume^p}{V} = volume\ fraction$$

Phase Average Modified Classical Lamination Theory

Assuming

$$(\varepsilon)_k^p = (\varepsilon)_k$$

main source error of approx.

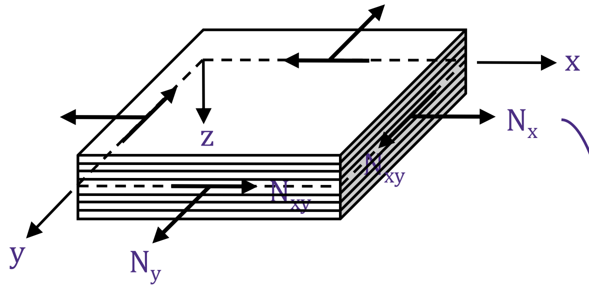
$$(\varepsilon)_k = (\varepsilon^o) + z_k(\kappa)$$

$$(\sigma)_k^p = [\bar{Q}]_k^p\left((\varepsilon)_k - (\alpha)_k^p \Delta T\right)$$

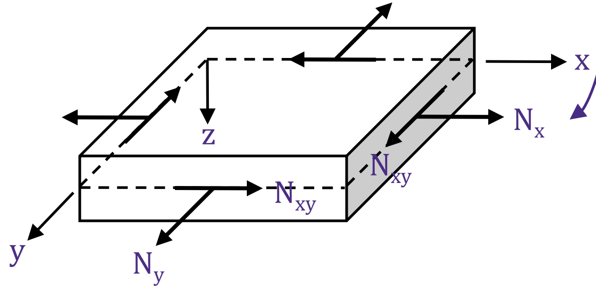
$$(\sigma)_k = \sum_{p=1}^{N} (\sigma)_k^p \, V_k^p$$

$$(N) = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} (\sigma)_k \, dz$$

$$(N) = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \sum_{p=1}^{N} [\bar{Q}]_k^p \left((\varepsilon^o) + z_k(\kappa) - (\alpha)_k^p \Delta T\right) V_k^p \, dz$$



homogenization



$$(N) = [A](\varepsilon^o) + [B]\,(\kappa) - (N_t)$$

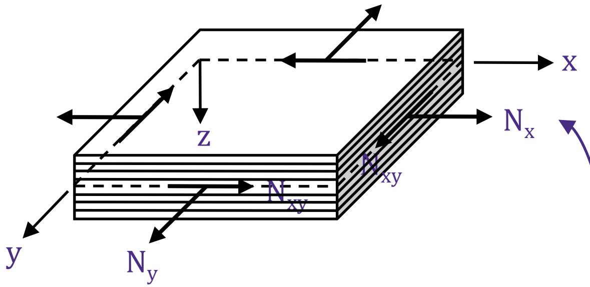
the homogenized stiffnesses of a N~phase laminated plate are;

$$[A] = \sum_{k=1}^{n}\left(\sum_{p=1}^{N} [\bar{Q}]_k^p \, V_k^p\right)(z_k - z_{k-1})$$

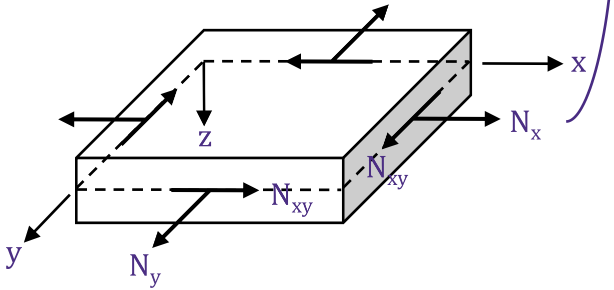
$$(N_t) = \Delta T \sum_{k=1}^{n}\left(\sum_{p=1}^{N} [\bar{Q}]_k^p (\alpha)_k^p \, V_k^p\right)(z_k - z_{k-1})$$

$$[B] = \frac{1}{2}\sum_{k=1}^{n}\left(\sum_{p=1}^{N} [\bar{Q}]_k^p \, V_k^p\right)(z_k^2 - z_{k-1}^2)$$

Phase Average Modified Classical Lamination Theory



De-homogenization



the stresses of within the p^{th} phase of the k^{th} ply are;

$$(\sigma)_k^p = [\bar{Q}]_k^p \left((\varepsilon)_k - (\alpha)_k^p \Delta T\right)$$

$$(\varepsilon)_k = (\varepsilon^o) + z_k(\kappa)$$

$$\begin{pmatrix} \varepsilon^o \\ \kappa \end{pmatrix} = \begin{bmatrix} a & b \\ b & d \end{bmatrix} \left(\begin{pmatrix} N \\ M \end{pmatrix} + \begin{pmatrix} N_t \\ M_t \end{pmatrix} \right)$$

$$\begin{pmatrix} N \\ M \end{pmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{pmatrix} \varepsilon^o \\ \kappa \end{pmatrix} - \begin{pmatrix} N_t \\ M_t \end{pmatrix}$$

starting from the solution to the homogenized laminated plate equations

Phase Average Modified Classical Lamination Theory

Classical

$$[A] = \sum_{k=1}^{n} [\bar{Q}]_k \, (z_k - z_{k-1})$$

$$[B] = \frac{1}{2} \sum_{k=1}^{n} [\bar{Q}]_k \, (z_k^2 - z_{k-1}^2)$$

$$[D] = \frac{1}{3} \sum_{k=1}^{n} [\bar{Q}]_k \, (z_k^3 - z_{k-1}^3)$$

$$(N_t) = \Delta T \sum_{k=1}^{n} [\bar{Q}]_k \, (\alpha)_k \, (z_k - z_{k-1})$$

$$(M_t) = \Delta T \frac{1}{2} \sum_{k=1}^{n} [\bar{Q}]_k \, (\alpha)_k \, (z_k - z_{k-1})$$

Phase Modified

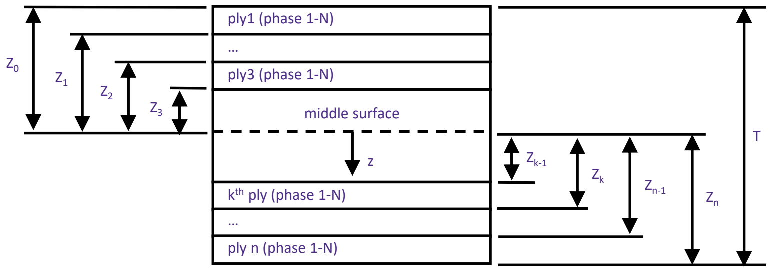
$$[A] = \sum_{k=1}^{n} \left(\sum_{p=1}^{N} [\bar{Q}]_k^p \, V_k^p \right) (z_k - z_{k-1})$$

$$[B] = \frac{1}{2} \sum_{k=1}^{n} \left(\sum_{p=1}^{N} [\bar{Q}]_k^p \, V_k^p \right) (z_k^2 - z_{k-1}^2)$$

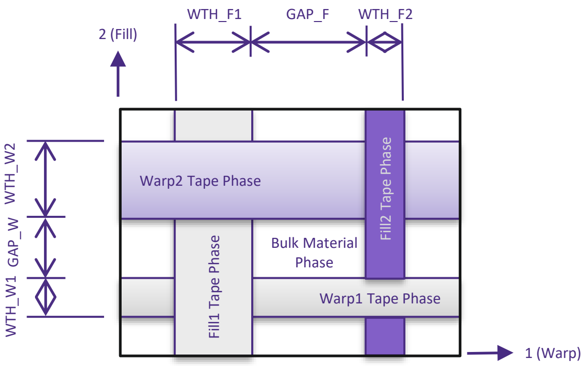
$$[D] = \frac{1}{3} \sum_{k=1}^{n} \left(\sum_{p=1}^{N} [\bar{Q}]_k^p \, V_k^p \right) (z_k^3 - z_{k-1}^3)$$

$$(N_t) = \Delta T \sum_{k=1}^{n} \left(\sum_{p=1}^{N} [\bar{Q}]_k^p (\alpha)_k^p \, V_k^p \right) (z_k - z_{k-1})$$

$$(M_t) = \Delta T \frac{1}{2} \sum_{k=1}^{n} \left(\sum_{p=1}^{N} [\bar{Q}]_k^p (\alpha)_k^p \, V_k^p \right) (z_k - z_{k-1})$$



$$\begin{pmatrix} N \\ M \end{pmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{pmatrix} \varepsilon^o \\ \kappa \end{pmatrix} - \begin{pmatrix} N_t \\ M_t \end{pmatrix}$$



Lattice Ply Layer

FEA Implementation & Numerical Validation

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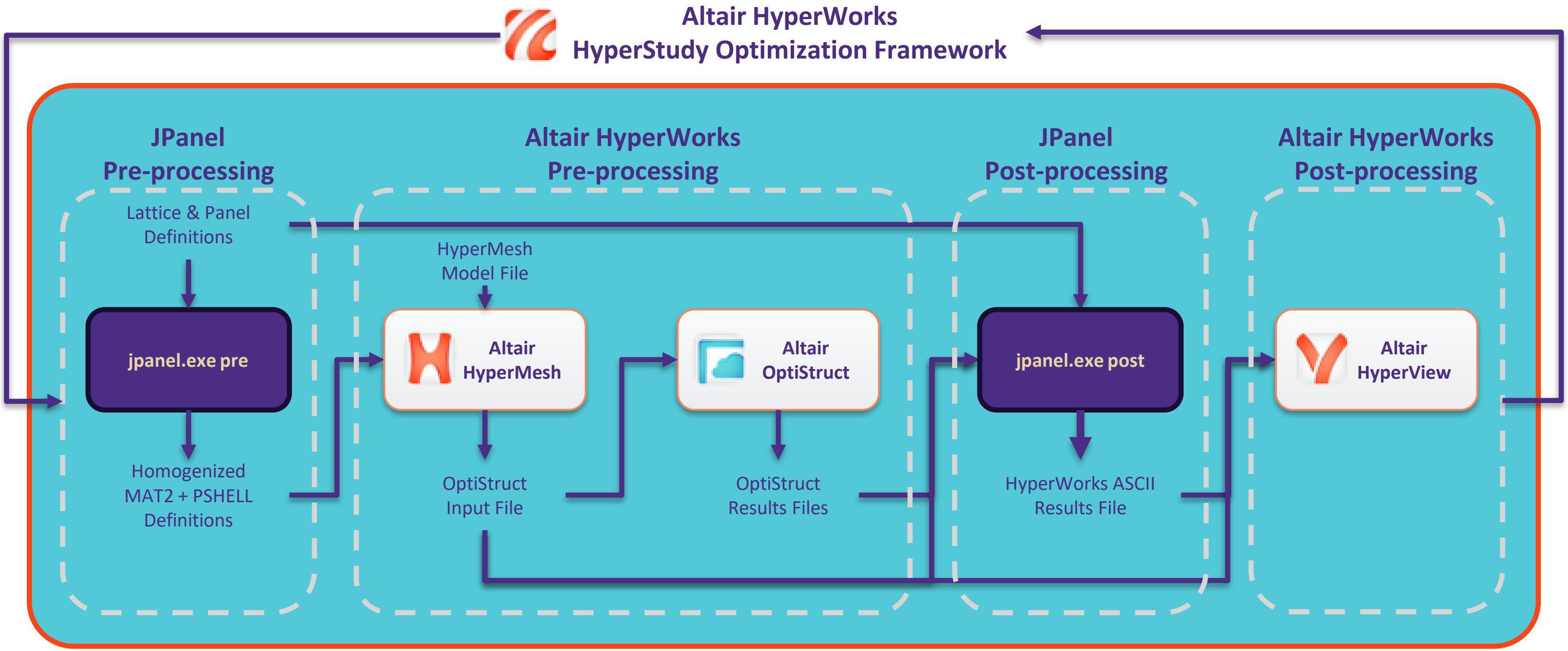
FEA Implementation & Architecture

Currently implemented in Altair HyperWorks (HyperMesh, OptiStruct, HyperView)



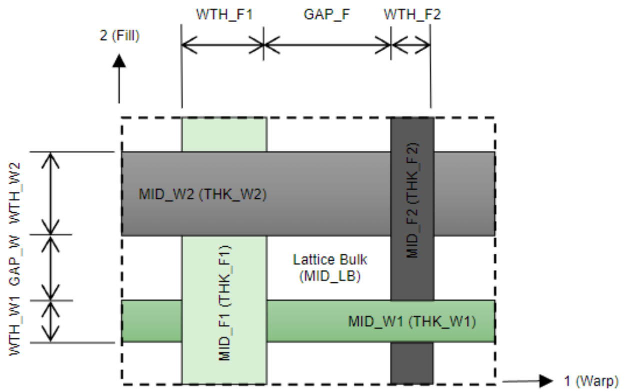
JPanel Input - Lattice + Panel Definitions

Individual Tape Material Properties (Warp1, Warp2, Fill1, Fill2, Bulk)

(1)	(2) / (6)	(3) / (7)	(4) / (8)	(5) / (9)	(10)
MAT9OR*	MID	E1	E2	E3	MNAME
	NU12	NU23	NU31	RHO	
	G12	G23	G13	A1	
	A2	A3			

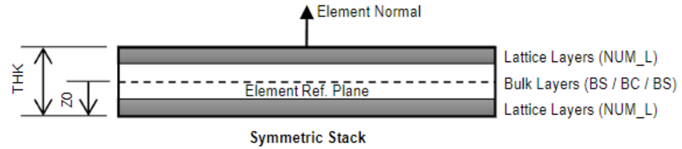
WEAV3D Lattice Definitions

(1)	(2) / (6)	(3) / (7)	(4) / (8)	(5) / (9)	(10)
LATTICE*	LID	LNAME	GAP_W	GAP_F	
	MID_W1	THK_W1	NUM_W1	WTH_W1	
	MID_W2	THK_W2	NUM_W2	WTH_W2	
	MID_F1	THK_F1	NUM_F1	WHT_F1	
	MID_F2	THK_F2	NUM_F2	WHT_F2	
	MID_LB				



Panel Definitions

(1)	(2) / (6)	(3) / (7)	(4) / (8)	(5) / (9)	(10)
PPANEL*	PID	PNAME	TYPE		
	LID	NUM_L	SYM	BOT	
	THK	Z0	NSM	PHI	
	MID_BC	MID_BS	VOL_BS		



JPanel OUTPUT – MAT2 + PSHELL

Captures the Membrane, Bending, Transverse Shear, and Coupling Behavior

(1)	(2) / (6)	(3) / (7)	(4) / (8)	(5) / (9)	(10)
MAT2*	MID	Q11	Q12	Q13	
	Q22	Q23	Q33	RHO	
	A1	A2	A12		

Membrane

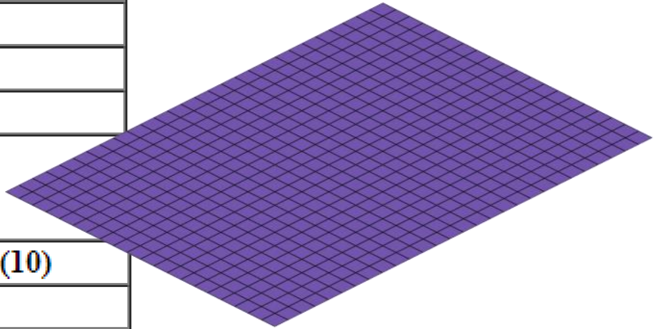
Bending

(1)	(2) / (6)	(3) / (7)	(4) / (8)	(5) / (9)	(10)
PSHELL*	PID	MID1	THK	MID2	
	12I/T3	MID3	TS/T	NSM	
	Z1	Z2	MID4		

Transverse Shear

Coupling

Multiscale Model



Numerical Validation

Material Properties

Glass/PP Uni Tape V^f = 45%

Property	Gpa	Psi
E_1	36.00	5.220e6
E_2	5.000	0.725e6
ν_{12}	0.300	0.300
ν_{23}	0.520	0.520
G_{12}	1.760	0.255e6
G_{23}	1.650	0.240e6
α_1	5.4e-6 /°C	3.0e-6 /°F
α_2	36.0e-6 /°C	20.0e-6 /°F
CPT	0.350mm	0.0138”
ρ	1.650 g/cm^3	0.0596 lbs/in^3

Carbon/PP Uni Tape V^f = 50%

Property	GPa	psi
E_1	110.0	15.950e6
E_2	4.800	0.696e6
ν_{12}	0.320	0.320
ν_{23}	0.480	0.480
G_{12}	2.000	0.290e6
G_{23}	1.620	0.235e6
α_1	-0.54e-6 /°C	-0.30e-6 /°F
α_2	40.0e-6 /°C	22.22e-6 /°F
CPT	0.160mm	0.0063”
ρ	1.310 g/cm^3	0.0473 lbs/in^3

Polypropylene (PP) Polymer

Property	GPa	psi
E	1.950	0.2828e6
ν	0.380	0.380
G	0.70652	0.1025e6
α	54.0e-6 /°C	30.0e-6 /°F
ρ	0.906 g/cm^3	0.0327 lbs/in^3

Numerical Validation

Lattice Geometry Extreme Case

Warp Geometry

V^{w1} = 20% (Glass/PP Tape)

V^{w2} = 40% (Carbon/PP Tape)

V^{wb} = 20% (PP Polymer Bulk)

Fill Geometry

V^{f1} = 28.57% (Carbon/PP Tape)

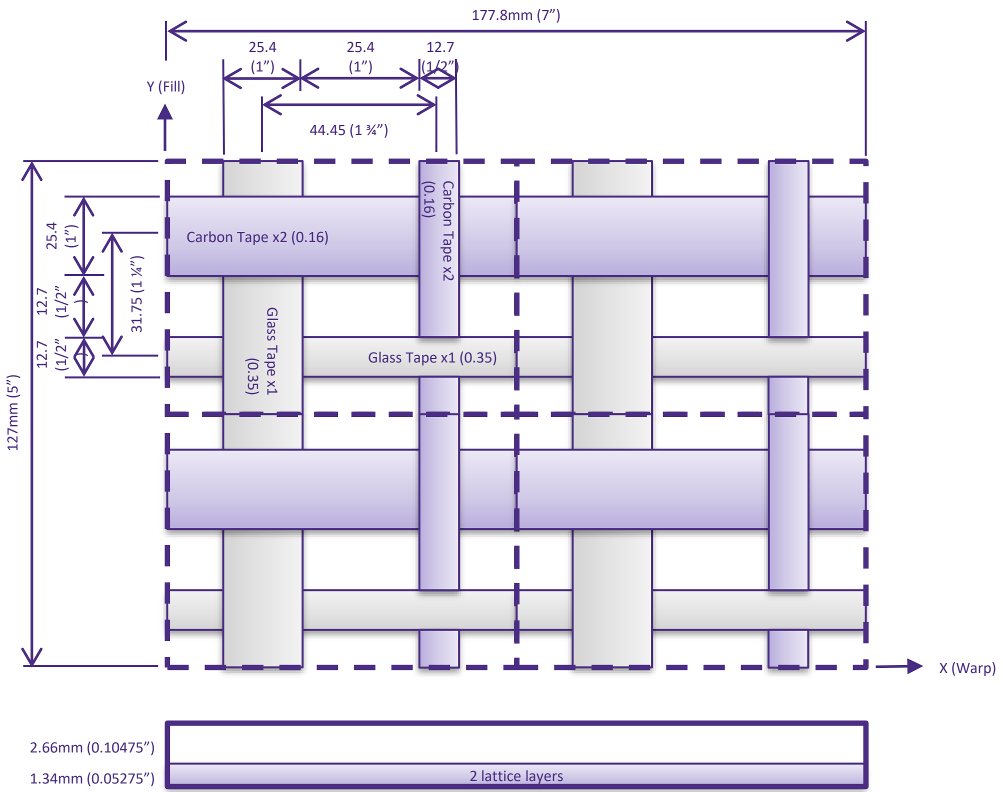
V^{f2} = 14.29% (Glass/PP Tape)

V^{fb} = 57.14% (PP Polymer Bulk)

Panel Geometry

2 lattice layers bottom only

Unsymmetric in-plane and through-thickness



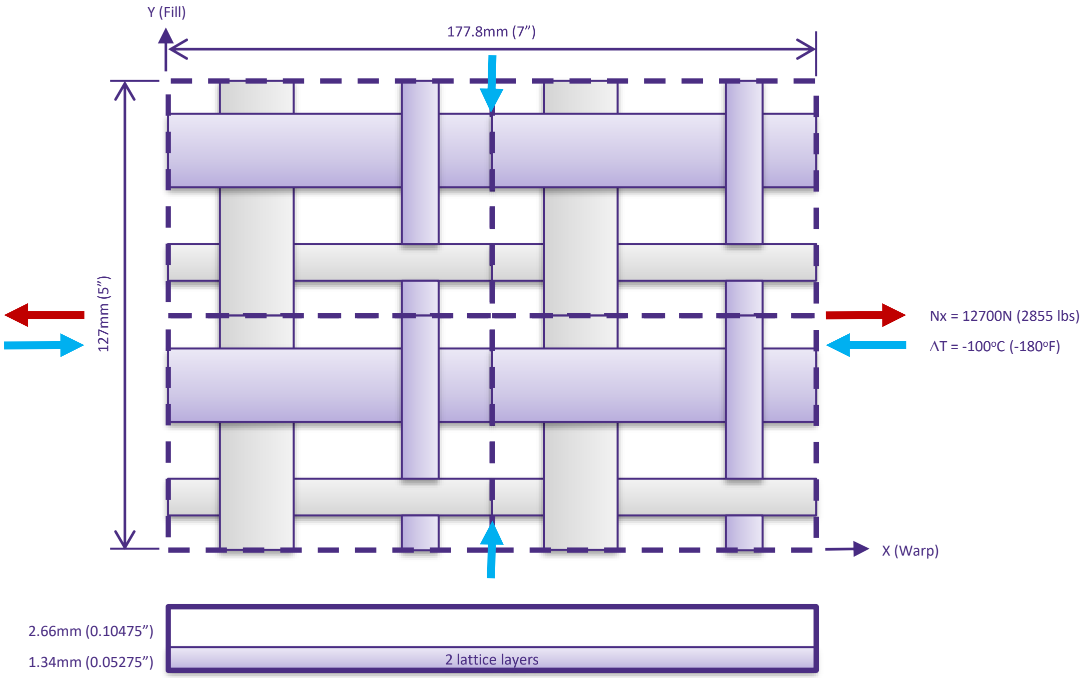
Numerical Validation

In-plane Load (Nx Case) and a Thermal Processing Load (ΔT Case)

Unsymmetric In-plane and through-thickness

Exercises In-plane and In-plane/Bending Coupling Behavior of Panels

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{14} & B_{11} & B_{12} & B_{14} \\ A_{12} & A_{22} & A_{24} & B_{12} & B_{22} & B_{24} \\ A_{14} & A_{24} & A_{44} & B_{14} & B_{24} & B_{44} \\ B_{11} & B_{12} & B_{14} & D_{11} & D_{12} & D_{14} \\ B_{12} & B_{22} & B_{24} & D_{12} & D_{22} & D_{24} \\ B_{14} & B_{24} & B_{44} & D_{14} & D_{24} & D_{44} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} - \begin{Bmatrix} N_{t,x} \\ N_{t,y} \\ N_{t,xy} \\ M_{t,x} \\ M_{t,y} \\ M_{t,xy} \end{Bmatrix}$$



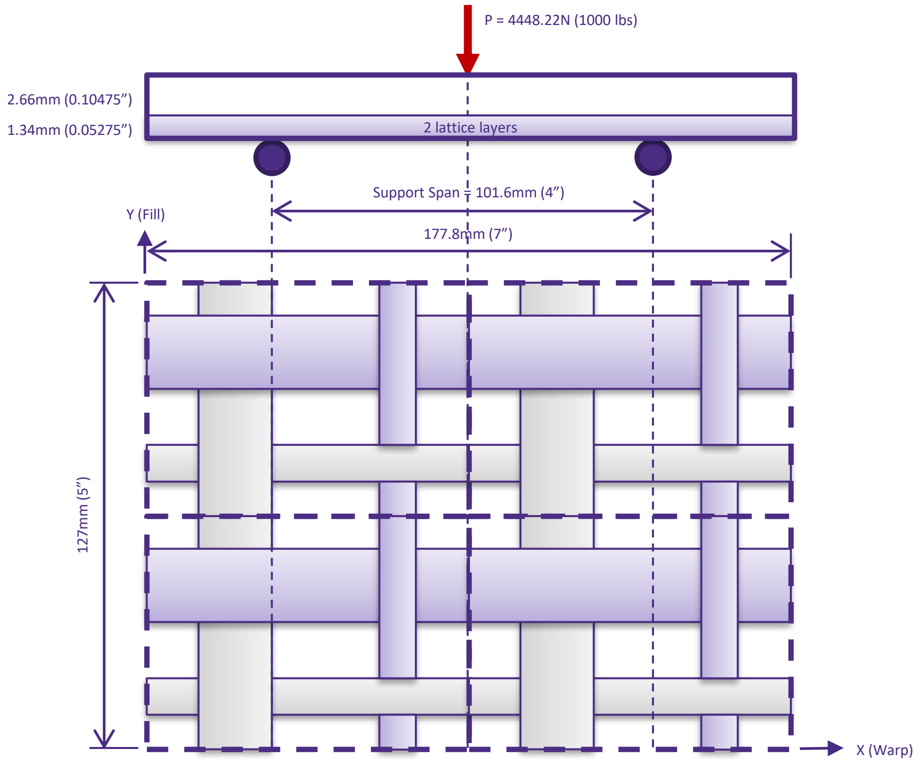
Numerical Validation

Bending Load (Mx Case)

Unsymmetric In-plane and through-thickness

Exercises Bending and Bending/In-Plane Coupling Behavior of Panels

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ \boxed{M_x} \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{14} & B_{11} & B_{12} & B_{14} \\ A_{12} & A_{22} & A_{24} & B_{12} & B_{22} & B_{24} \\ A_{14} & A_{24} & A_{44} & B_{14} & B_{24} & B_{44} \\ B_{11} & B_{12} & B_{14} & D_{11} & D_{12} & D_{14} \\ B_{12} & B_{22} & B_{24} & D_{12} & D_{22} & D_{24} \\ B_{14} & B_{24} & B_{44} & D_{14} & D_{24} & D_{44} \end{bmatrix} \boxed{\begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}} - \begin{Bmatrix} N_{t,x} \\ N_{t,y} \\ N_{t,xy} \\ M_{t,x} \\ M_{t,y} \\ M_{t,xy} \end{Bmatrix}$$



Discrete vs Multiscale Model

Discrete Modeling

-Explicitly model features
-typically most Accurate Results
-can be Difficult to Model
-can be Computationally Expensive

VS

Multiscale Modeling

-Implicitly model features
-can achieve Accurate Results
-significantly Easier Modeling
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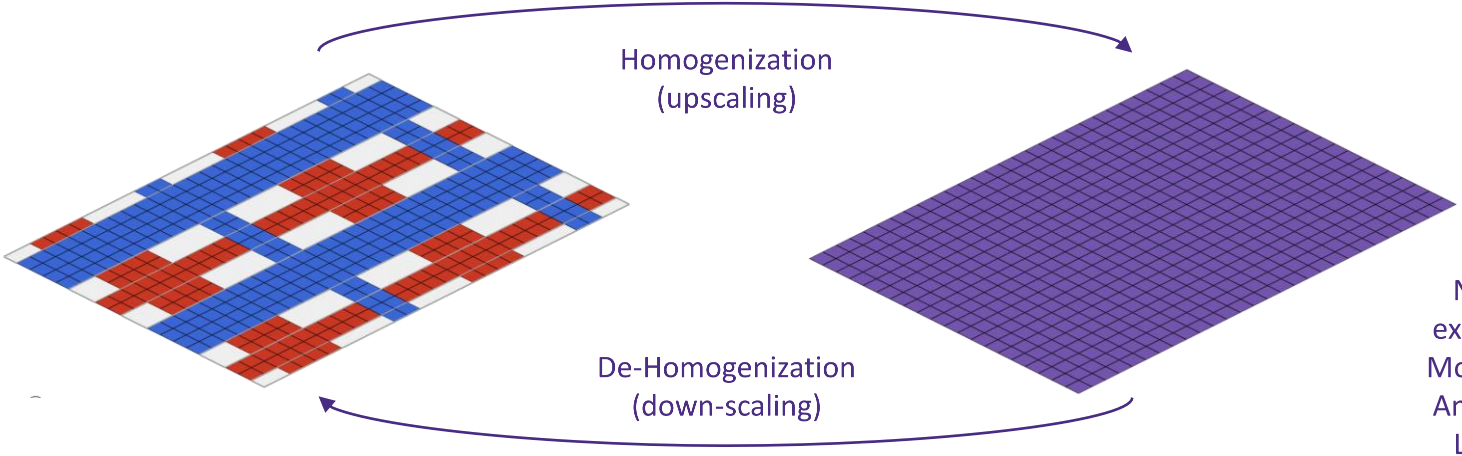
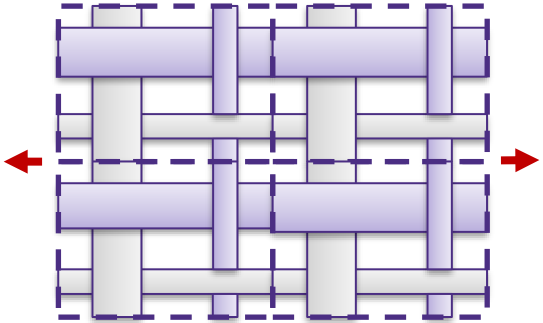


Note: A great early example of Multiscale Modeling is Composite Analysis with Classical Lamination Theory

Numerical Validation - Displacements

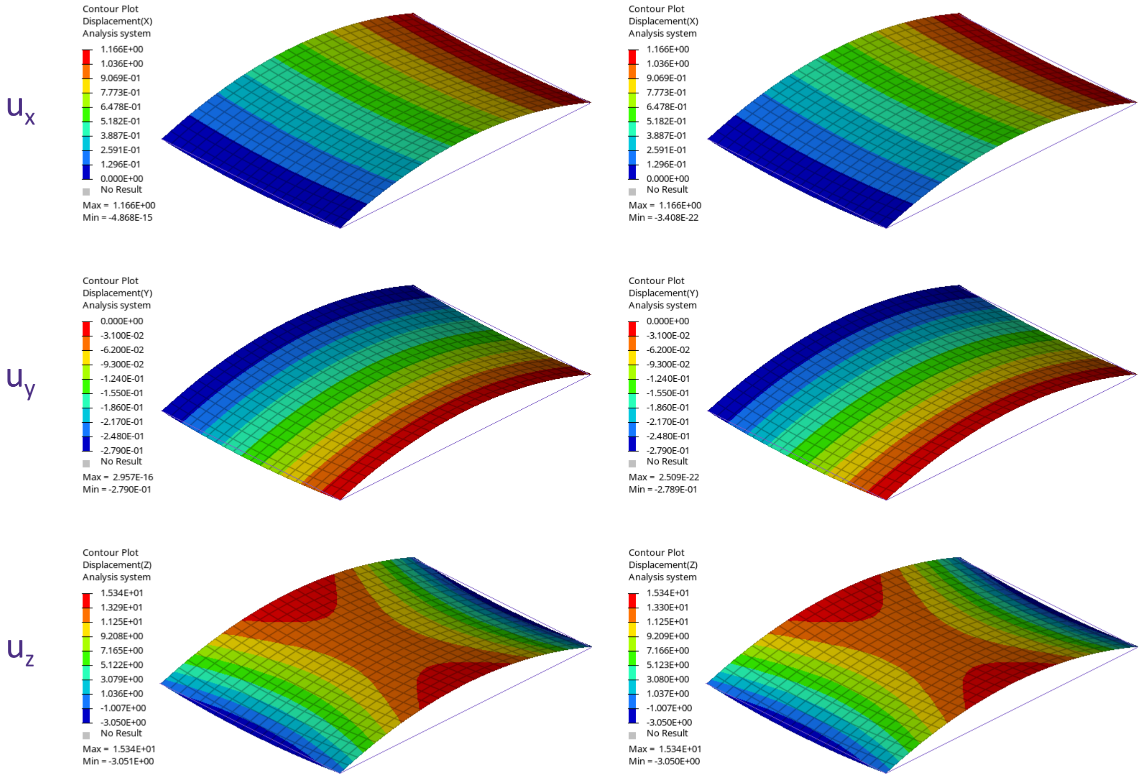
Nx Case

Correctly bends “up” and all displacements within ±1% error of approximation



Discrete Model

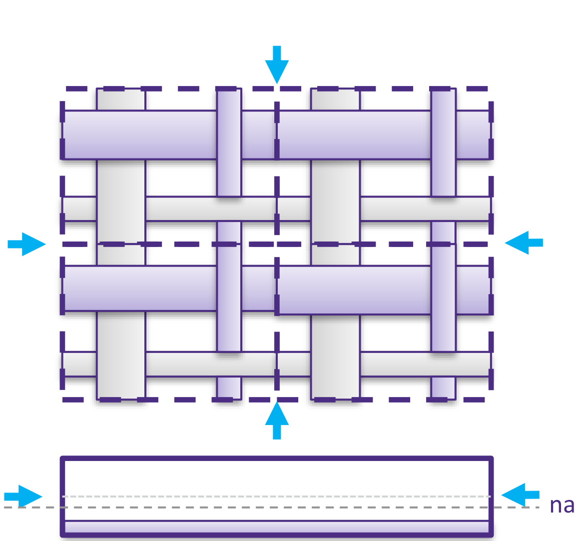
Multiscale Model



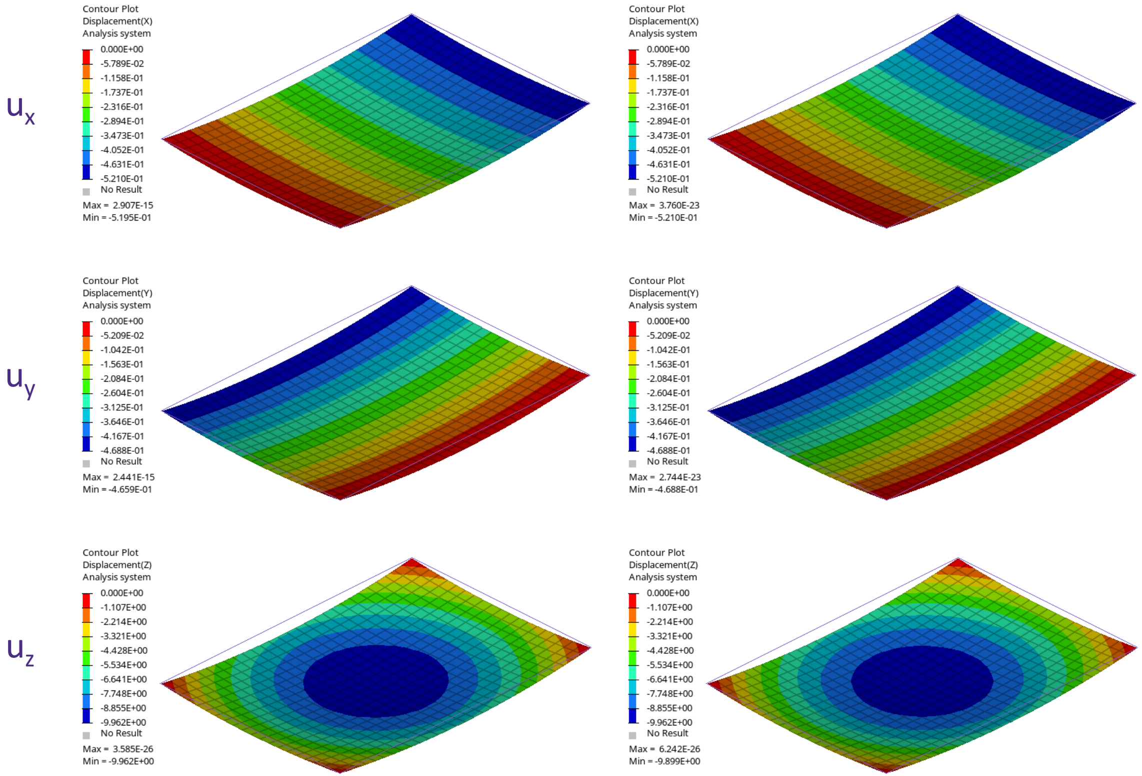
Numerical Validation - Displacements

Δx Case

Correctly bends “down” and all displacements within ±1% error of approximation



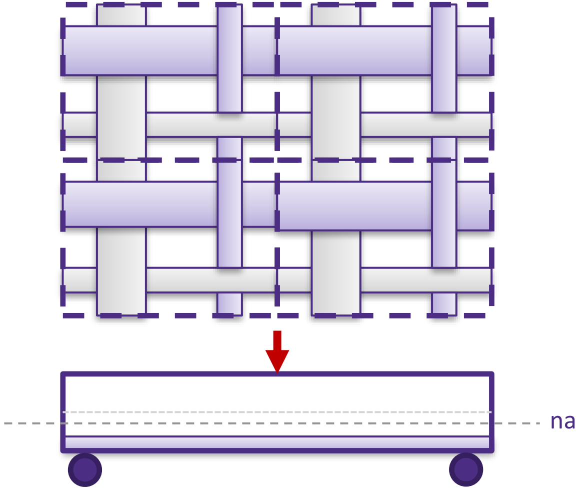
Discrete Model | **Multiscale Model**



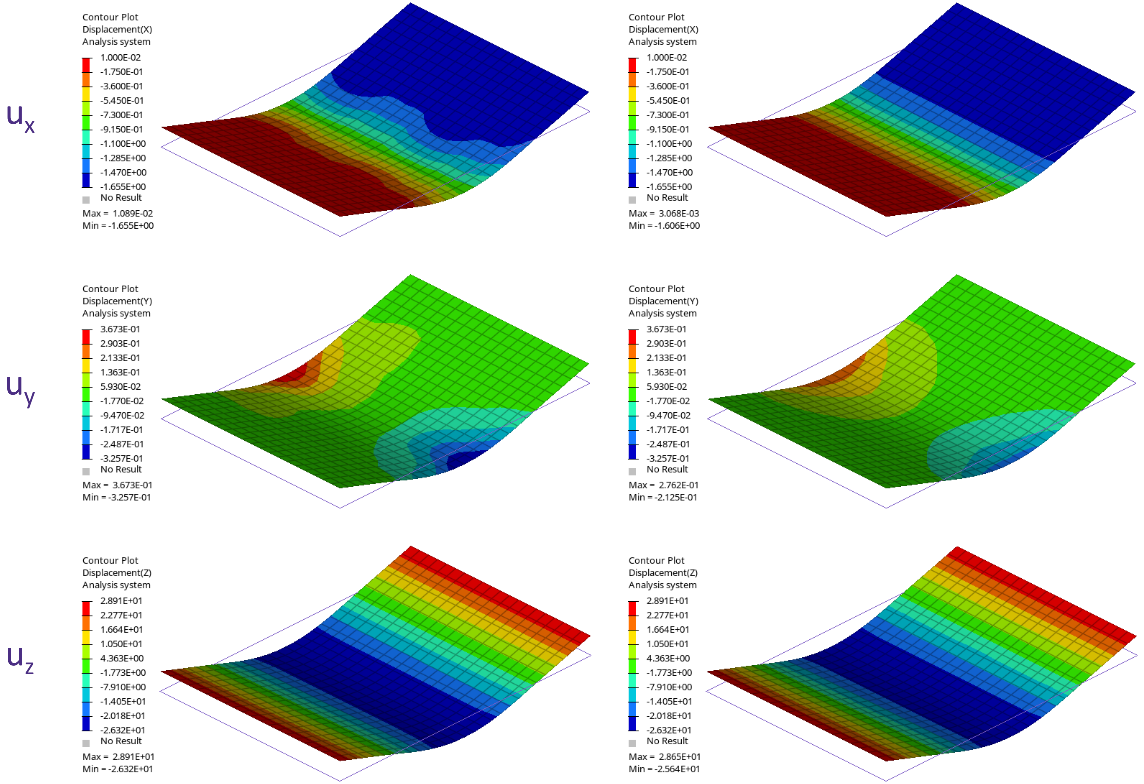
Numerical Validation - Displacements

Mx Case

The in-plane unsymmetric geometry (extreme case) is not completely accounted for and causes increased but acceptable in-plane error but out-of-plane error still within ±1% error of approximation



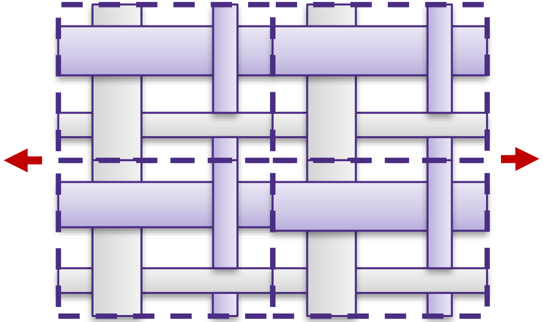
Discrete Model | **Multiscale Model**

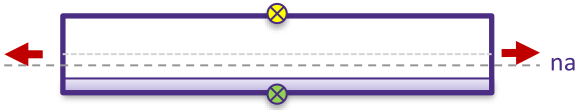


Numerical Validation - Stresses

Nx Case

The primary axial stress in each phase are captured and within ±10% error of approximation

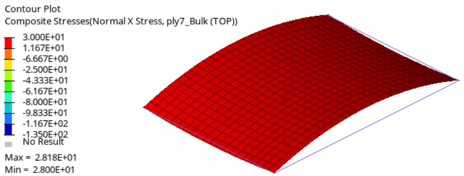


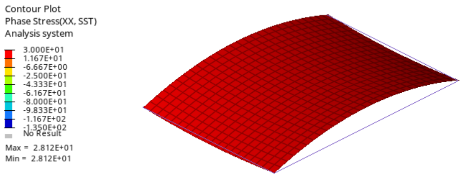


Discrete Model

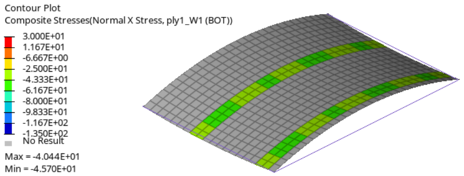
Multiscale Model

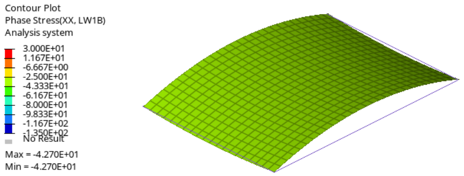
$\sigma_x^{\text{panel bulk}}$ (PP)



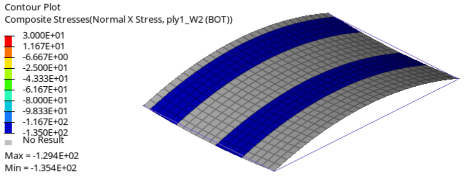


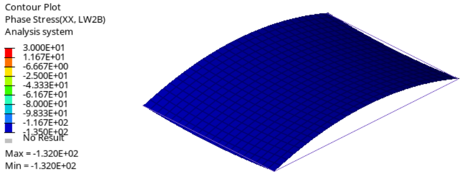
$\sigma_x^{\text{lattice warp1}}$ (Glass/PP)



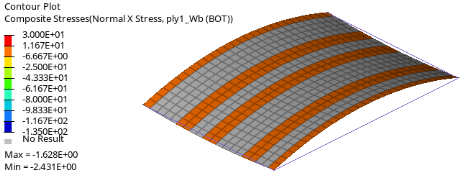


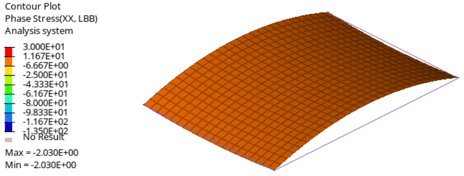
$\sigma_x^{\text{lattice warp2}}$ (Carbon/PP)





$\sigma_x^{\text{lattice bulk}}$ (PP)

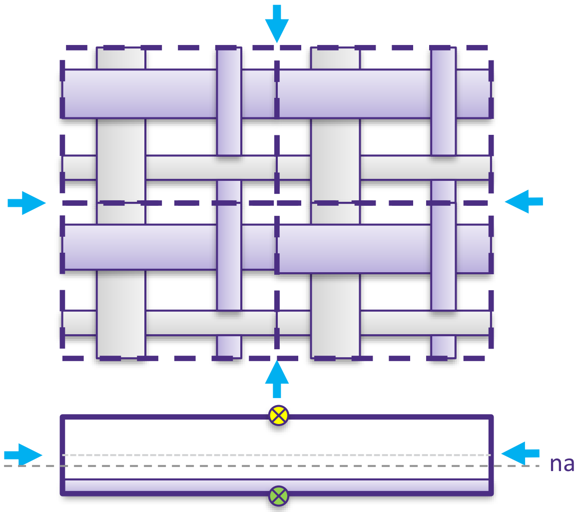




Numerical Validation - Stresses

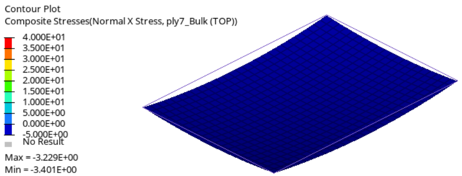
Δx Case

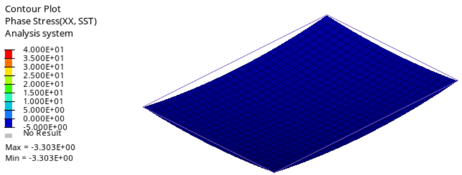
The thermal residual axial stress in each phase are captured and within ±10% error of approximation



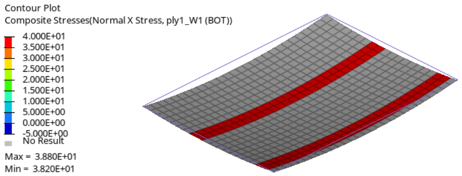
Discrete Model | Multiscale Model

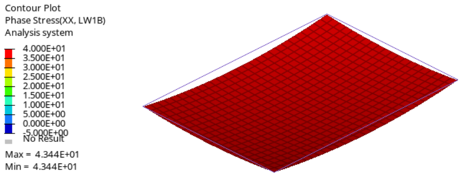
$\sigma_x^{\text{panel bulk}}$ (PP)



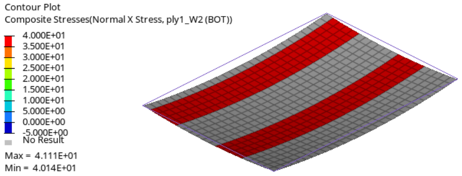


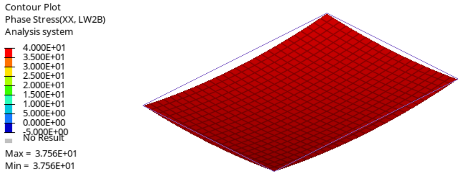
$\sigma_x^{\text{lattice warp1}}$ (Glass/PP)



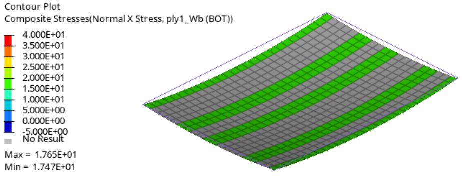


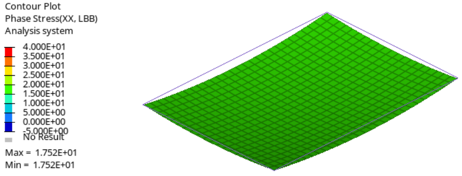
$\sigma_x^{\text{lattice warp2}}$ (Carbon/PP)





$\sigma_x^{\text{lattice bulk}}$ (PP)

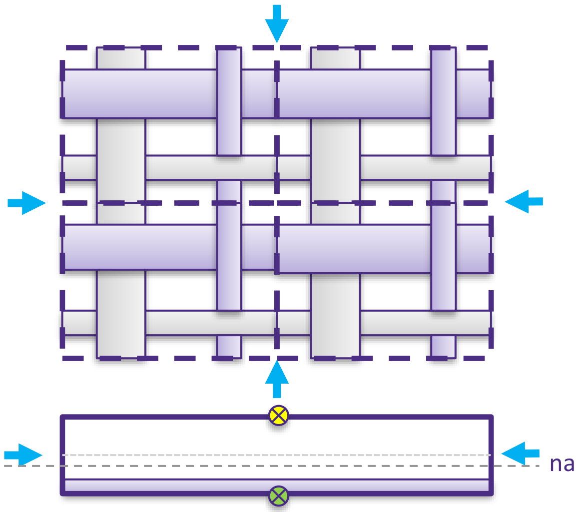




Numerical Validation - Stresses

Δx Case

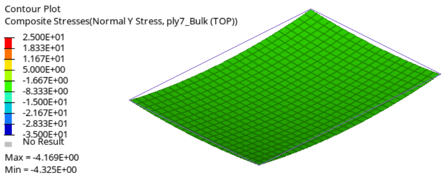
The thermal residual transverse stress in each phase are captured and within ±10% error of approximation

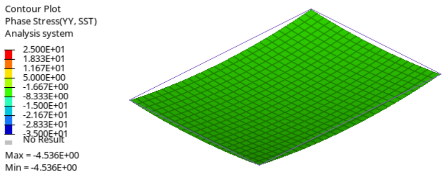


Discrete Model

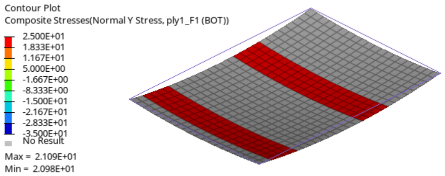
Multiscale Model

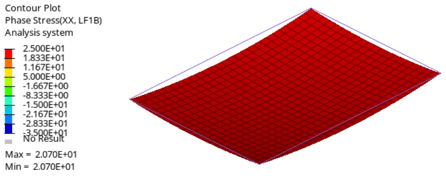
$\sigma_x^{\text{panel bulk}}$ (PP)



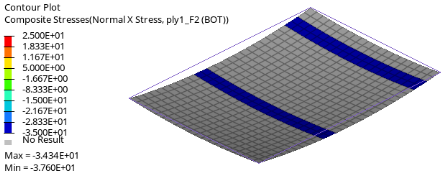


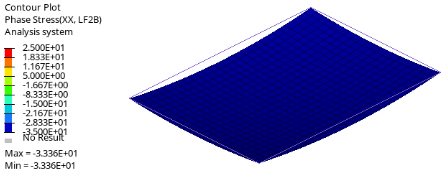
$\sigma_x^{\text{lattice warp1}}$ (Glass/PP)



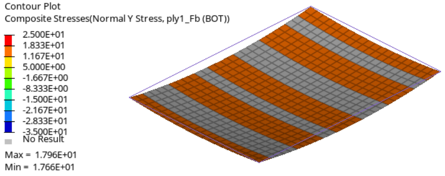


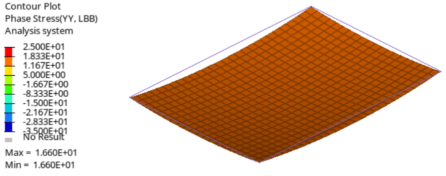
$\sigma_x^{\text{lattice warp2}}$ (Carbon/PP)





$\sigma_x^{\text{lattice bulk}}$ (PP)

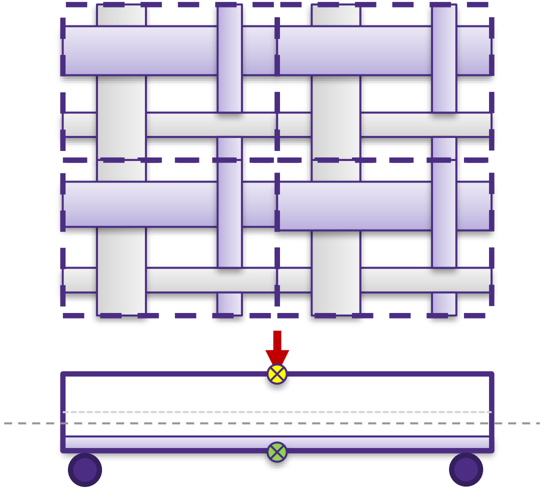




Numerical Validation - Stresses

Mx Case

The primary axial stress in bending in each phase are captured and within ±10% error of approximation



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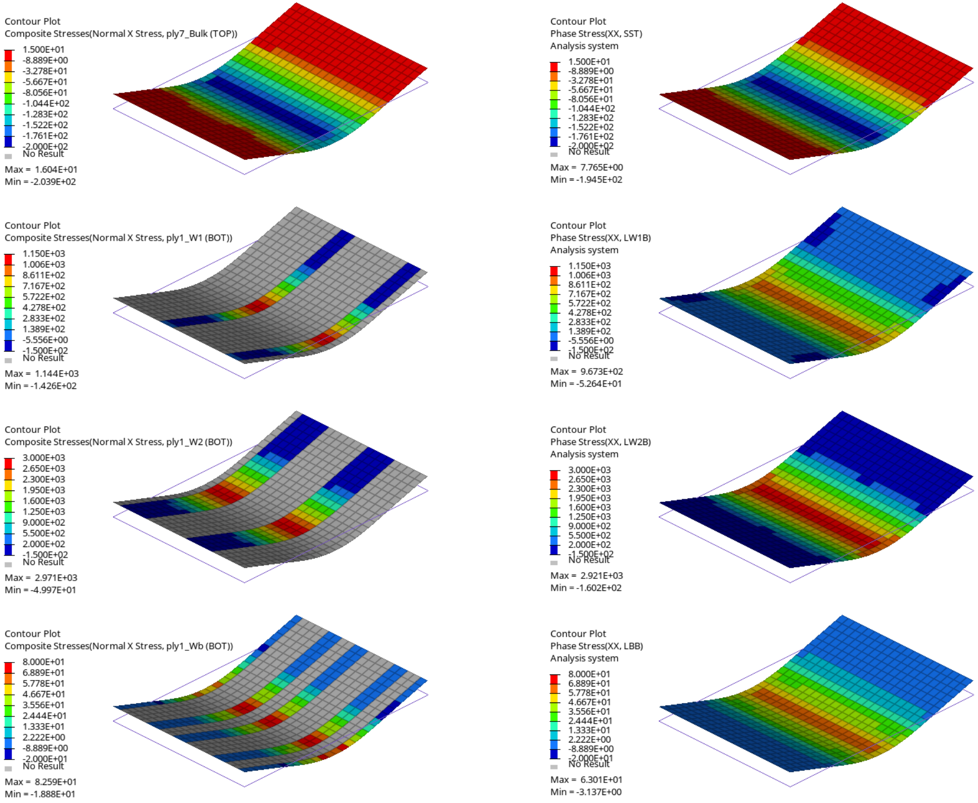
Discrete Model | Multiscale Model

$\sigma_x^{\text{panel bulk}}$ (PP)

$\sigma_x^{\text{lattice warp1}}$ (Glass/PP)

$\sigma_x^{\text{lattice warp2}}$ (Carbon/PP)

$\sigma_x^{\text{lattice bulk}}$ (PP)



Conclusions

- Discrete Modeling Methods for the Design and Analysis of WEAV3D Lattice Structures is impractical for general use cases
- Multiscale Methods appear to be a reasonable solution to the challenges of analyzing and designing WEAV3D Lattice Structures
- The error of approximation of such approaches appears to be within acceptable engineering error tolerances of ±10%
- The significant reduction in effort to create and apply the multiscale modeling, as opposed to discrete modeling, and given acceptable error of approximation, warrants the use multiscale modeling for this problem…

Continuing Project Phases

BE BOUNDLESS

Continuing Forward

1. Experimental Test Program for Tape Properties
2. Experimental Test Program for 3pt Bend Panels
3. Numerical Verification (comparison of simulation with experiment as opposed to discrete model presented here)
4. Optimization Methodology & Software
5. An Optimization Example with Verification

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